Using the Membership Table Method (MTM) to Determine the Validity of Categorical Syllogisms

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October 30, 2021

Abstract

A presentation is provided of a method – the Membership Table Method (MTM) – to determine the validity of categorical syllogisms. This method makes it possible for each syllogism to be assigned a specific set. If this set is equal to the universal set $\mathbb{U}$, then the categorical syllogism considered is valid, and if that set is not equal to $\mathbb{U}$, then that categorical syllogism is not valid. In other words, any categorical syllogism is valid if and only if its respective set, according to the MTM, is equal to the universal set $\mathbb{U}$. The conclusion of a valid categorical syllogism whose premises are true is true.

Keywords: propositional calculus, set theory, categorical syllogisms, truth tables, membership tables, membership table method

Mathematics Subject Classification 2020: 03B05, 97E60, 03E20, 03B10, 97E30

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1 Introduction

From historical and educational standpoints, forms of reasoning known as syllogisms, especially categorical syllogisms, have been relevant in the development of logic.

The objective of this article is to present a method to determine the validity of categorical syllogisms: the membership table method (MTM). It can be applied more simply and systematically than other pre-existing methods. Up until now one of the most commonly used approaches to determine the validity of categorical syllogisms has been that of using diverse types of diagrams [1], [2], [3], [4], [5], and [6]. A previous contribution by the authors of this article, with that objective, can be classified within that approach [7].

A clear understanding of this article requires only a basic knowledge of the propositional calculus of classical logic and of set theory. For an introduction to classical logic, one may consult, for example: [8], [9], [10], [11], and [12]. On set theory, one may consult, for example, [13], [14], [15], and [16].

2 A Correspondence between Operations of Propositional Calculus and Operations of Set Theory

Propositional variables – that is, variables that can be replaced with propositions – are usually referred to as p, q, r, etc. Given that in other articles related to this one the letter p will be used to refer to a probability and diverse probabilities will be denominated p_1, p_2, p_3, etc., to prevent confusion, in this article these propositional variables will be symbolized as follows: q_1, q_2, q_3, etc. (with some exceptions as specified in section 5). In addition, taking advantage of a "license" used by a number of authors, q_1, q_2, q_3, etc. will be referred to as "propositions". Thus, statements such as "q_1 is true" and "q_2 is false" should be interpreted respectively in the following way: "Admit that q_1 has been replaced with a true proposition" and "Admit that q_2 has been replaced with a false proposition". If a sole proposition is considered, the subscript can be eliminated and the proposition symbolized as q.

In set theory, the universal set, or the universe of discourse, to which all the elements that may be considered when referring to a given topic belong, will be symbolized as $\mathbb{U}$.

The diverse sets characterized within the framework of the universal set $\mathbb{U}$ will be denominated C_1, C_2, C_3, etc. (with some exceptions to be specified in section 6). If only one of those sets is considered, the subscript can be eliminated and the set symbolized as C.

The (monadic) logical connective of negation in propositional calculus will be symbolized by a horizontal bar above the negated proposition. Thus, the symbol $\overline{q}$ – that is, not q – will represent the negation of that proposition q.

All the elements of the set $\mathbb{U}$ considered that do not belong to C belong to the complement of the set C. The complement of C is symbolized as $\overset{+}{C}$. In this article the symbol $+$ corresponding to the operator of complementation of a set is placed above its operand. This operand is the set C on which that operator acts, thus generating the complement of C (that is, $\overset{+}{C}$), which is another set.

Given that all of the elements which may be considered when dealing with a particular topic belong to the set $\mathbb{U}$, no element belongs to the complement of $\mathbb{U}$, known as the empty set. It is symbolized as $\varnothing$.

The relation between the universal set $\mathbb{U}$ and the corresponding empty set can be expressed by the following equality:

$$\overset{+}{\mathbb{U}} = \varnothing \tag{1}$$

Given that a) all the elements belonging to $\mathbb{U}$ that do not belong to C belong to the complement of any set C and that b) no element of the corresponding $\mathbb{U}$ belongs to the set $\varnothing$, it is obvious that

$$\overset{+}{\varnothing} = \mathbb{U} \tag{2}$$

Given that because of (2), $\overset{+}{\varnothing}$ and $\mathbb{U}$ are the same set, their complements must also be equal:

$$\overset{+}{\overset{+}{\varnothing}} = \overset{+}{\mathbb{U}} \tag{3}$$

If, in (3), $\overset{+}{\mathbb{U}}$ is replaced with $\varnothing$, admissible given the equality (1), the following equation is obtained:

$$\overset{+}{\overset{+}{\varnothing}} = \varnothing \tag{4}$$

In addition, given that because of (1), $\overset{+}{\mathbb{U}}$ and $\varnothing$ are the same set, their complements will also be equal:

$$\overset{+}{\overset{+}{\mathbb{U}}} = \overset{+}{\varnothing} \tag{5}$$

According to (2), $\overset{+}{\varnothing}$ can be replaced in (5) with $\mathbb{U}$. Thus the following equation is obtained:

$$\overset{+}{\overset{+}{\mathbb{U}}} = \mathbb{U} \tag{6}$$

In general, the complement of the complement of any set C is equal to C:

$$\overset{+}{\overset{+}{C}} = C \tag{7}$$

The truth tables (a) of q, $\overline{q}$ and $\overline{\overline{q}}$, and the membership tables (b) of C, $\overset{+}{C}$ and $\overset{+}{\overset{+}{C}}$ are presented in figure 1.

q	$\overline{q}$	$\overline{\overline{q}}$
0	1	0
1	0	1

(a) Truth tables of $\overline{q}$ (that is, of the negation of q) and of $\overline{\overline{q}}$ (that is, of the negation of the negation of q, or of the "double negation" of q).

C	$\overset{+}{C}$	$\overset{+}{\overset{+}{C}}$
0	1	0
1	0	1

(b) Membership tables of $\overset{+}{C}$ (that is, of the complement of C) and of $\overset{+}{\overset{+}{C}}$ (that is, of the complement of the complement of C).

Figure 1: a) Truth tables of $\overline{q}$ and $\overline{\overline{q}}$, and b) membership tables of $\overset{+}{C}$ and $\overset{+}{\overset{+}{C}}$.

In figure 1a, the presence of a 0 in the column corresponding to a proposition, such as q, means that the proposition is considered false. The presence of a 1 in the column corresponding to a proposition means that the proposition is considered true. Classical logic is bivalent, in the sense that there are only two possible truth values for any proposition: It is true or it is false. In that bivalent logic, the negation of any true proposition is a false proposition, and the negation of any false proposition is a true proposition.

The first row of the truth tables represented in figure 1a is composed of the numerical sequence $0, 1, 0$. It should be interpreted in the following way: If the proposition q is false, then its negation $\overline{q}$ is true and the negation of $\overline{q}$ (that is, $\overline{\overline{q}}$) is true.

Note in figure 1a that the columns corresponding to q and $\overline{\overline{q}}$ are identical: asserting q is equivalent to asserting $\overline{\overline{q}}$.

In figure 1b, the set C mentioned above is any set characterized within the framework of any universal set $\mathbb{U}$; that is, any element belonging to C is an element belonging to that $\mathbb{U}$.

In figure 1b, the presence of a 0 in a column corresponding to a set, such as C, means the supposition that any element belonging to $\mathbb{U}$ does not belong to that set. The presence of a 1 in the column corresponding to a set means the supposition that any element belonging to $\mathbb{U}$ does belong to that set. In classic set theory, if any element of $\mathbb{U}$ belongs to C then that element does not belong to $\overset{+}{C}$, and if that element of $\mathbb{U}$ does not belong to C then that element does belong to $\overset{+}{C}$.

The first row of the membership tables in figure 1b is composed of the numerical sequence 0, 1, 0. It should be interpreted as follows: If any element of $\mathbb{U}$ (that is, belonging to $\mathbb{U}$) does not belong to C, then that element does belong to the complement of C (i.e., $\overset{+}{C}$) and does not belong to the complement of $\overset{+}{C}$ (i.e., $\overset{+}{\overset{+}{C}}$).

The second row of the membership tables in figure 1b is composed of the numerical sequence 1, 0, 1. It should be interpreted as follows: If any element of $\mathbb{U}$ belongs to C, then that element does not belong to the complement of C (i.e., $\overset{+}{C}$) and does belong to the complement of $\overset{+}{C}$ (i.e., $\overset{+}{\overset{+}{C}}$).

Note that the columns corresponding to C and to $\overset{+}{\overset{+}{C}}$ are identical: $C = \overset{+}{\overset{+}{C}}$, as seen in (7); the double operation of complementation of a set results in the set C itself.

The first and the second rows of the truth tables in figure 1a are the same, respectively, as the first and second rows of the membership tables in figure 1b. That makes it possible to establish a) the correspondence of the proposition q with the set C, b) the correspondence of the proposition $\overline{q}$ with the set $\overset{+}{C}$, c) the correspondence of the proposition $\overline{\overline{q}}$ with the set $\overset{+}{\overset{+}{C}}$, and d) the correspondence of the connective of negation in propositional calculus with the operator of complementation in set theory.

The logical connective of conjunction of two propositions is symbolized as $\wedge$. The proposition $q_1 \wedge q_2$ – the conjunction of the propositions q_1 and q_2 – is read as: "q_1 and q_2".

The operator of the intersection of two sets is symbolized as $\cap$. The set which is the intersection of the sets C_1 and C_2 – two sets characterized within the framework of some universal set $\mathbb{U}$ – is symbolized as $C_1 \cap C_2$.

Figure 2 presents a) the truth table of the proposition $q_1 \wedge q_2$, and b) the membership table of the set $C_1 \cap C_2$.

q_1	q_2	$q_1 \wedge q_2$
0	0	0
0	1	0
1	0	0
1	1	1

(a) Truth table of the proposition $q_1 \wedge q_2$.

C_1	C_2	$C_1 \cap C_2$
0	0	0
0	1	0
1	0	0
1	1	1

(b) Membership table of the set $C_1 \cap C_2$.

Figure 2: a) Truth table of $q_1 \wedge q_2$; and b) membership table of $C_1 \cap C_2$.

In the truth table of $q_1 \wedge q_2$ (in figure 2a), it is seen that only when both q_1 and q_2 are true, is $q_1 \wedge q_2$ also true. This case corresponds to the fourth row in that truth table, which is composed of the numerical sequence 1, 1, 1; if and only if q_1 and q_2 are true, is $q_1 \wedge q_2$ also true.

In the membership table $C_1 \cap C_2$ (in figure 2b), it is seen that only when any element of $\mathbb{U}$ belongs both to C_1 and to C_2, does that element belong to $C_1 \cap C_2$. This case corresponds to the fourth row of that membership table, which is composed of the numerical sequence 1, 1, 1; if and only if any element of $\mathbb{U}$ belongs both to C_1 and to C_2, does that element also belong to $C_1 \cap C_2$.

Note, in figure 2, the equality of the first, second, third and fourth rows, respectively, in figure 2a and figure 2b. This makes it possible to establish a correspondence between a) the propositions q_1 and q_2 and the sets C_1 and C_2, respectively; b) the proposition $q_1 \wedge q_2$ (the conjunction of q_1 and q_2) and the set $C_1 \cap C_2$ (the set which is the intersection of C_1 and C_2); and c) the logical connective of conjunction in propositional calculus ($\wedge$) and the operator of intersection in set theory ($\cap$).

The logical connective of disjunction of two propositions will be symbolized as $\vee$. The proposition $q_1 \vee q_2$ – the disjunction of the propositions q_1 and q_2 – is read as: "q_1 or q_2".

The operator of union of two sets is symbolized as $\cup$. The set which is the union of the sets C_1 and C_2 – two sets characterized within the framework of some universal set $\mathbb{U}$ – is symbolized as $C_1 \cup C_2$.

Figure 3 presents a) the truth table of the proposition $q_1 \vee q_2$, and b) the membership table of $C_1 \cup C_2$.

q_1	q_2	$q_1 \vee q_2$
0	0	0
0	1	1
1	0	1
1	1	1

(a) Truth table of the proposition $q_1 \vee q_2$.

C_1	C_2	$C_1 \cup C_2$
0	0	0
0	1	1
1	0	1
1	1	1

(b) Membership table of the set $C_1 \cup C_2$.

Figure 3: a) Truth table of $q_1 \vee q_2$; and b) membership table of $C_1 \cup C_2$.

Note in the truth table in figure 3a that only when both q_1 and q_2 are false (in the first row of that table) is $q_1 \vee q_2$ false. Note also in the membership table in figure 3b that only when any element of $\mathbb{U}$ belongs neither to C_1 nor to C_2 (in the first row of that table) does that element not belong to $C_1 \cup C_2$.

Note that the first, second, third and fourth rows in figure 3a are the same, respectively, as the first, second, third and fourth rows in 3b. Therefore, a correspondence can be established between a) the propositions q_1 and q_2 and the sets C_1 and C_2, respectively; b) the proposition $q_1 \vee q_2$ and the set $C_1 \cup C_2$;

and c) the logical connective of disjunction in propositional calculus and the operator of union in set theory.

The logical connective of material implication in propositional calculus will be represented as $\rightarrow$. The proposition $q_1 \rightarrow q_2$ is read as: "If q_1, then q_2". The proposition q_1 is known as the antecedent of $q_1 \rightarrow q_2$ and the proposition q_2 is known as the consequent of that proposition.

The operator of the material implication in set theory will be represented as $\mathrel{-\!\!\!+\!\!\!\rightarrow}$. The result of that operator acting on the ordered pair of sets $\{C_1, C_2\}$ is the set $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$.

The proposition $q_2 \rightarrow q_1$ is read as: "If q_2, then q_1". The proposition q_2 is known as the antecedent of $q_2 \rightarrow q_1$ and the proposition q_1 is known as the consequent of that proposition.

The result of the operator of material implication acting on the ordered pair of sets $\{C_2, C_1\}$ is equal to the set $C_2 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$.

Figure 4 presents a) the truth tables of the propositions $q_1 \rightarrow q_2$ and $q_2 \rightarrow q_1$; and b) the membership tables of the sets $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$ and $C_2 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$.

q_1	q_2	$q_1 \rightarrow q_2$	$q_2 \rightarrow q_1$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

(a) Truth tables of $q_1 \rightarrow q_2$ and $q_2 \rightarrow q_1$.

C_1	C_2	$C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$	$C_2 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

(b) Membership tables of $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$ and $C_2 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$.

Figure 4: a) Truth tables of the propositions $q_1 \rightarrow q_2$ and $q_2 \rightarrow q_1$, and membership tables of the sets $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$ and $C_2 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$.

Note in figure 4a that the only case in which the proposition $q_1 \rightarrow q_2$ is false (in the third row in the truth table of that proposition) is that in which the antecedent q_1 is true and the consequent q_2 is false. In the other three cases, $q_1 \rightarrow q_2$ is true. Note also in figure 4b, that the only case in which any element whatsoever of the universal set $\mathbb{U}$ does not belong to the set $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$ (in the third row of the membership table of that set) is that in which the element belongs to C_1 and does not belong to C_2. In the other three cases, that element does belong to $C_1 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_2$.

Note in figure 4a that the only case in which the proposition $q_2 \rightarrow q_1$ is false (in the second row in the truth table of that proposition) is that in which the antecedent q_2 is true and the consequent q_1 is false. In the other three cases,

$q_2 \to q_1$ is true. Note also in figure 4b that the only case in which any element whatsoever of the universal set $\mathbb{U}$ does not belong to the set $C_2 \twoheadrightarrow C_1$ (in the second row of the membership table of that set) is that in which that element belongs to C_2 and does not belong to C_1. In the other three cases, that element does belong to $C_2 \twoheadrightarrow C_1$.

Observe that the first, second, third and fourth rows of the truth table of $q_1 \to q_2$ are the same respectively, as the first, second, third and fourth rows of the membership table of $C_1 \twoheadrightarrow C_2$. Observe also that the first, second, third and fourth rows of the truth table of $q_2 \to q_1$ are the same, respectively, as the first, second, third and fourth rows of the membership table of $C_2 \twoheadrightarrow C_1$. Therefore, a correspondence may be established between a) the propositions q_1 and q_2 and the sets C_1 and C_2, respectively; b) the propositions $q_1 \to q_2$ and $q_2 \to q_1$ and the sets $C_1 \twoheadrightarrow C_2$ and $C_2 \twoheadrightarrow C_1$, respectively; and c) the logical connective of material implication in propositional calculus and the operator of material implication in set theory.

The logical connective of material bi-implication, or of logical equivalence in propositional calculus, will be represented as $\longleftrightarrow$. The proposition $q_1 \longleftrightarrow q_2$ is read as: "The proposition q_1 is logically equivalent to the proposition q_2". The operator of material bi-implication in set theory will be represented as $\twoheadleftarrow\!\twoheadrightarrow$.

Figure 5 presents a) the truth table of $q_1 \longleftrightarrow q_2$, and b) the membership table of the set $C_1 \twoheadleftarrow\!\twoheadrightarrow C_2$

q_1	q_2	$q_1 \longleftrightarrow q_2$
0	0	1
0	1	0
1	0	0
1	1	1

(a) Truth table of $q_1 \longleftrightarrow q_2$.

C_1	C_2	$C_1 \twoheadleftarrow\!\twoheadrightarrow C_2$
0	0	1
0	1	0
1	0	0
1	1	1

(b) Membership table of the set $C_1 \twoheadleftarrow\!\twoheadrightarrow C_2$.

Figure 5: a) Truth table of $q_1 \longleftrightarrow q_2$ and b) membership table of the set $C_1 \twoheadleftarrow\!\twoheadrightarrow C_2$.

In figure 5a it is seen that the two cases in which the proposition $q_1 \longleftrightarrow q_2$ is true are those (in the first and fourth rows of the truth table of that proposition) in which q_1 and q_2 have the same value of truth. In the case represented in the first row both q_1 and q_2 are false, and in the case represented in the fourth row both q_1 and q_2 are true.

In figure 5b it is seen that the two cases in which any element whatsoever of the universal set $\mathbb{U}$ considered belongs to the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$ are the following: a) in the case in which the element belongs neither to C_1 nor to C_2 (represented in the first row of the membership table of that set) and b) the case in which that element belongs both to C_1 and to C_2 (represented in the fourth row of the membership table of that set).

Observe that the first, second, third and fourth rows of the truth table of $q_1 \longleftrightarrow q_2$ are the same, respectively, as the first, second, third and fourth rows of the membership table of $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$. Thus a correspondence can be established between a) the propositions q_1 and q_2 and the sets C_1 and C_2, respectively; b) the proposition $q_1 \longleftrightarrow q_2$ and the set $C_1 \twoheadleftarrow\!\!\twoheadrightarrow C_2$; and c) the logical connective of material bi-implication in propositional calculus and the operator of material bi-implication in set theory.

For propositions resulting from the use of connectives, such as $q_1 \to q_2$ or $q_3 \vee q_4$, it can be suitable to express them in parentheses as $(q_1 \to q_2)$ and $(q_3 \vee q_4)$, respectively. Thus, if a connective is used with those propositions to obtain another proposition, it is clear how that has operated. For example, $(q_1 \to q_2) \to (q_3 \vee q_4)$ is the proposition of a conditional nature: "If $q_1 \to q_2$, then $q_3 \vee q_4$", in which $q_1 \to q_2$ is the antecedent, and $q_3 \vee q_4$ is the consequent. The proposition $(q_1 \to q_2) \to (q_3 \vee q_4)$ has been obtained by the action of the connective of material implication on the following ordered pair of propositions $\{(q_1 \to q_2), (q_3 \vee q_4)\}$. Likewise, for clarity, the proposition $(q_1 \to q_2) \to (q_3 \vee q_4)$ can be expressed in parentheses if an operation is carried out on it and on some other proposition, by using some logical connective. Hence, for example, $((q_1 \to q_2) \to (q_3 \vee q_4)) \wedge (q_1 \to q_5)$ is the proposition obtained through the action of the logical connective of conjunction on the propositions $(q_1 \to q_2) \to (q_3 \vee q_4)$ and $(q_1 \to q_5)$.

Given the correspondences mentioned, a) between propositions and sets, and b) between logical connectives and set theory operators, considerations of this same type concerning the use of parentheses are valid in that theory. Therefore, the set $(C_1 \twoheadrightarrow C_2) \twoheadrightarrow (C_3 \cup C_4)$ corresponds to $(q_1 \to q_2) \to (q_3 \vee q_4)$, the set $C_1 \twoheadrightarrow C_5$ corresponds to the proposition $q_1 \to q_5$, and the set $((C_1 \twoheadrightarrow C_2) \twoheadrightarrow (C_3 \cup C_4)) \cap (C_1 \twoheadrightarrow C_5)$ corresponds to the proposition $((q_1 \to q_2) \to (q_3 \vee q_4)) \wedge (q_1 \to q_5)$.

If in the logical operations carried out there are n propositions – q_1, q_2, $\ldots, q_n$ – in the corresponding truth tables, there will be 2^n rows because each of those propositions can have two truth values: true or false. Each row of those tables will correspond to each possible case of different assignments for the truth values of each of those n propositions. Since for each of these cases there are two possible assignments of truth value for the logical function to be specified, there are $2^{(2^n)}$ possible logical functions of n propositions. Thus, for example, if $n = 2$, there are 16 possible logical functions; if $n = 3$, there are 256 possible logical functions; and if $n = 4$, there are 65,536 possible logical functions.

For each of the $2^{(2^n)}$ logical functions of n propositions, for $n = 1, 2, 3 \ldots$, there is, according to the approach used, a function of n sets, which also is a set.

3 Isomorphism between Each Law – Theorem or Tautology – of Propositional Calculus and the Corresponding Expression of the Universal Set

This section will provide only a) the characterizations of the main notions concerning the topic discussed; and b) three examples of isomorphism existing between the tautologies of propositional calculus and the corresponding sets (according to section 2).

If a function of n propositions, for $n = 1, 2, 3, \ldots$, is true regardless of the truth values of each of those n propositions, then that function, which is also a proposition, is considered a law – or tautology – of propositional calculus. A tautology is true given its logical form, or structure.

The negation of a tautology which is a function of n propositions is known as a contradiction and is false, regardless of the values of truth of each of those n propositions.

The law of propositional calculus known in Latin as *modus tollendo ponens* – that is, "the mode that, by denying, affirms"– is stated below in (8); and the corresponding set, which is isomorphic to it according to section 2, in (9).

$$((q_1 \vee q_2) \wedge \overline{q}_2) \rightarrow q_1 \tag{8}$$

$$((C_1 \cup C_2) \cap \overset{+}{C}_2) \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1 \tag{9}$$

Figure 6 presents a) the truth table of the proposition specified in (8), and b) the membership table of the corresponding set specified in (9).

q_1	q_2	$q_1 \vee q_2$	$\overline{q}_2$	$(q_1 \vee q_2) \wedge \overline{q}_2$	$((q_1 \vee q_2) \wedge \overline{q}_2) \rightarrow q_1$
0	0	0	1	0	1
0	1	1	0	0	1
1	0	1	1	1	1
1	1	1	0	0	1

(a) Truth table of the proposition specified in (8)

C_1	C_2	$C_1 \cup C_2$	$\overset{+}{C}_2$	$(C_1 \cup C_2) \cap \overset{+}{C}_2$	$((C_1 \cup C_2) \cap \overset{+}{C}_2) \mathrel{-\!\!\!+\!\!\!\rightarrow} C_1$
0	0	0	1	0	1
0	1	1	0	0	1
1	0	1	1	1	1
1	1	1	0	0	1

(b) Membership table of the corresponding set, specified in (9)

Figure 6: a) Truth table of the proposition specified in (8), and b) membership table of the corresponding set, specified in (9)

In the truth table in figure 6a it can be seen that the proposition $((q_1 \vee q_2) \wedge \overline{q}_2) \rightarrow q_1$ is a tautology because it is true given its logical form; that is, it is true regardless of the truth values of q_1 and q_2.

In the membership table in 6b it can be seen that for any element of the universal set $\mathbb{U}$ there are four possibilities: 1) that it belongs neither to C_1 nor to C_2 (as in the first row of the membership table); 2) that it does not belong to C_1 and does belong to C_2 (as in the second row of the membership table); 3) that it belongs to C_1 and not to C_2 (as in the third row of the membership table); and 4) that it belongs both to C_1 and to C_2 (as in the fourth row of the membership table). In each of these cases, the element of $\mathbb{U}$ considered belongs to the set $((C_1 \cup C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} C_1$. Therefore, given that any element of $\mathbb{U}$ belongs to that set, it is inferred that the set is equal to $\mathbb{U}$: $((C_1 \cup C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} C_1 = \mathbb{U}$.

The law of propositional calculus known in Latin as *modus tollendo tollens* – that is, the mode that by denying, denies – is stated in (10) below; and the corresponding set, which is isomorphic to it, is stated in (11).

$$((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1 \tag{10}$$

$$((C_1 \mathrel{+\!\!\!\!\twoheadrightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} \overset{+}{C}_1 \tag{11}$$

Figure 7 presents a) the truth table of the proposition specified in (10) and b) the membership table of the corresponding set, isomorphic to the proposition.

q_1	q_2	$q_1 \rightarrow q_2$	$\overline{q}_2$	$(q_1 \rightarrow q_2) \wedge \overline{q}_2$	$\overline{q}_1$	$((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$
0	0	1	1	1	1	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

(a) Truth table of the proposition specified in (10)

C_1	C_2	$C_1 \rightarrow C_2$	$\overset{+}{C}_2$	$(C_1 \mathrel{+\!\!\!\!\twoheadrightarrow} C_2) \cap \overset{+}{C}_2$	$\overset{+}{C}_1$	$((C_1 \mathrel{+\!\!\!\!\twoheadrightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} \overset{+}{C}_1$
0	0	1	1	1	1	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

(b) Membership table of the set specified in (11)

Figure 7: a) Truth table of the proposition specified in (10), and b) membership table of the set specified in (11)

In figure 7 it is seen a) that $((q_1 \rightarrow q_2) \wedge \overline{q}_2) \rightarrow \overline{q}_1$ is a tautology of propositional calculus, and b) that the corresponding set $((C_1 \mathrel{+\!\!\!\!\twoheadrightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} \overset{+}{C}_1$, which is isomorphic to it, is equal to the universal set $\mathbb{U}$: $(C_1 \mathrel{+\!\!\!\!\twoheadrightarrow} C_2) \cap \overset{+}{C}_2) \mathrel{+\!\!\!\!\twoheadrightarrow} \overset{+}{C}_1$.

A law of propositional calculus – one of the laws of De Morgan – is stated in (12), and the corresponding set, which is isomorphic to it, is stated in (13) below.

$$\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2) \tag{12}$$

$$\overline{(C_1 \overset{+}{\cap} C_2)} \leftarrow\!\!+\!\!\rightarrow (\overset{+}{C}_1 \cup \overset{+}{C}_2) \tag{13}$$

Figure 8 presents a) the truth table of the proposition specified in (12) and b) the membership table of the set specified in (13).

q_1	q_2	$q_1 \wedge q_2$	$\overline{(q_1 \wedge q_2)}$	$\overline{q}_1$	$\overline{q}_2$	$(\overline{q}_1 \vee \overline{q}_2)$	$\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2)$
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

(a) Truth table of the proposition specified in (12)

C_1	C_2	$C_1 \cap C_2$	$\overline{(C_1 \overset{+}{\cap} C_2)}$	$\overset{+}{C}_1$	$\overset{+}{C}_2$	$(\overset{+}{C}_1 \cup \overset{+}{C}_2)$	$\overline{(C_1 \overset{+}{\cap} C_2)} \leftarrow\!\!+\!\!\rightarrow (\overset{+}{C}_1 \cup \overset{+}{C}_2)$
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

(b) Membership table of the set specified in (13)

Figure 8: a) Truth table of the proposition specified in (12), and b) membership table of the set specified in (13)

In figure 8 it is seen a) that $\overline{(q_1 \wedge q_2)} \longleftrightarrow (\overline{q}_1 \vee \overline{q}_2)$ is a tautology of propositional calculus and b) that the corresponding set $\overline{(C_1 \overset{+}{\cap} C_2)} \leftarrow\!\!+\!\!\rightarrow (\overset{+}{C}_1 \cup \overset{+}{C}_2)$, which is isomorphic to it, is equal to the universal set $\mathbb{U}$: $\overline{(C_1 \overset{+}{\cap} C_2)} \leftarrow\!\!+\!\!\rightarrow (\overset{+}{C}_1 \cup \overset{+}{C}_2) = \mathbb{U}$.

4 Categorical Propositions and Their Corresponding Sets

Categorical propositions are assertions about two sets that affirm or deny that one of those sets is totally or partially included in the other set. In this section those sets are called C_4 and C_5 respectively. Those names have been chosen for the following reason: When referring to categorical syllogisms, reasoning of a particular type, the names C_1, C_2 and C_3 will be used to refer systematically to certain sets which play an important role in that reasoning. To avoid confusion, those three names are not used in this section. Of course, instead of using C_4 and C_5, names such as F and G respectively could have been used.

Four examples of categorical propositions are given below:

1. All generals are brave.
2. No generals are brave.
3. Some generals are brave.
4. Some generals are not brave.

In each of the four categorical propositions, reference is made to the following two sets:

C_4: generals; C_5: brave persons

The following is another set of four examples of categorical propositions:

1. All philosophers are honest.
2. No philosophers are honest.
3. Some philosophers are honest.
4. Some philosophers are not honest.

The sets referred to in each of these other four categorical propositions are these:

C_4: philosophers; C_5: honest persons

If both sets of categorical propositions are compared, it is noted that each proposition of the first set has the same logical form as the proposition to which the same number was assigned in the second set. Thus, four types of categorical propositions can be distinguished. Any categorical proposition, given its logical form, may be ascribed to one of these four types.

The propositions assigned number 1, in both sets of four propositions, are examples of those known as affirmative universal propositions. The symbolic expression of either of them is the following:

All C_4 are C_5.

According to the preceding proposition, if any element whatsoever of the universal set $\mathbb{U}$ considered belongs to the set C_4, then it belongs also to the set C_5. If that proposition is true, then the possibility that any element whatsoever of $\mathbb{U}$ could belong to C_4 and not also to C_5 is eliminated. According to section 2, any element of $\mathbb{U}$ belonging to the set $C_4 \mathrel{-\!\!+\!\!\!\to} C_5$ satisfies that condition; according to the membership table of $C_4 \mathrel{-\!\!+\!\!\!\to} C_5$, the possibility that the element could belong to C_4 and not to C_5 is eliminated.

The symbolic expression of any affirmative universal categorical proposition is stated once more below, with the corresponding set at the right.

All C_4 are C_5. $\qquad\qquad (C_4 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_5)$

The propositions assigned number 2, in both sets of four propositions, are examples of those called negative universal propositions. The symbolic expression of either of them is the following:

No C_4 are C_5.

According to the preceding proposition, if any element of the universal set $\mathbb{U}$ considered belongs to the set C_4, then it does not belong to the set C_5; that is, it also does belong to the set $\overset{+}{C}_5$. If that proposition is true, then the possibility of any element whatsoever of $\mathbb{U}$ belonging to C_4, but not belonging also to $\overset{+}{C}_5$, is eliminated. Any element of $\mathbb{U}$ belonging to the set $(C_4 \mathrel{-\!\!\!+\!\!\!\rightarrow} \overset{+}{C}_5)$ satisfies this condition: According to the membership table of $(C_4 \mathrel{-\!\!\!+\!\!\!\rightarrow} C_5)$, the possibility of that element belonging to C_4, but not belonging also to $\overset{+}{C}_5$, is eliminated.

Below the symbolic expression of any negative universal categorical proposition is given once more, with the corresponding set on the right.

No C_4 are C_5. $\qquad\qquad (C_4 \mathrel{-\!\!\!+\!\!\!\rightarrow} \overset{+}{C}_5)$

The propositions assigned number 3, in both sets of propositions, are examples of those called affirmative particular categorical propositions. The symbolic expression of either of them is the following:

Some C_4 are C_5.

According to the preceding proposition there is at least one element of the universal set $\mathbb{U}$ considered which belongs both to C_4 and to C_5. If there is not at least one element of $\mathbb{U}$ that belongs both to C_4 and to C_5, then that proposition is false. According to section 2, any element of $\mathbb{U}$ that belongs both to C_4 and to C_5 belongs to the set which is the intersection of C_4 and C_5 – $C_4 \cap C_5$. Thus, $C_4 \cap C_5$ is the set corresponding to that proposition.

The symbolic expression of any affirmative particular categorical proposition is stated again below, with the corresponding set at the right.

Some C_4 are C_5. $\qquad\qquad (C_4 \cap C_5)$

The propositions assigned the number 4, in both sets of four propositions, are examples of those known as negative particular categorical propositions. The symbolic expression of either of them is the following:

Some C_4 are not C_5.

According to the preceding proposition, there is at least one element of the universal set $\mathbb{U}$ that belongs to C_4 and does not belong to C_5; that is, it belongs to $\overset{+}{C}_5$. If there is not at least one element of $\mathbb{U}$ that belongs both to C_4 and to $\overset{+}{C}_5$, then that proposition is false. Any element of $\mathbb{U}$ that belongs both to C_4 and to $\overset{+}{C}_5$ belongs to the set that is the intersection of C_4 and $\overset{+}{C}_5$ – $C_4 \cap \overset{+}{C}_5$. Thus, $C_4 \cap \overset{+}{C}_5$ is the set corresponding to that proposition.

The symbolic expression of any particular negative categorical proposition is given below once more, with the corresponding set at the right.

Some C_4 are not C_5. $\qquad\qquad$ $(C_4 \cap \overset{+}{C}_5)$

5 Characterization of Categorical Syllogisms

A syllogism is a type of deductive reasoning in which a categorical proposition known as a conclusion is inferred, or deduced, from two other categorical propositions known as premises.

In this article the first premise of each categorical syllogism will be named s_1, given that "*s*tatement" or "*s*entence" are considered synonyms of "proposition". The second premise and the conclusion will be called s_2 and s_3, respectively. These denominations have been preferred to q_1, q_2 and q_3, for example, to emphasize that these are not just any three propositions, but rather three categorical propositions constituting a categorical syllogism, linked together due to their internal structures.

The predicate term of the conclusion – s_3 – is known as the major term of the syllogism and the subject term of s_3 is called the minor term of the syllogism. The major premise of the syllogism, which in this article will correspond systematically to s_1, is that which contains the major term. The minor premise of the syllogism, which in this article will correspond to s_2, is that which contains the minor term.

The third term of the syllogism does not appear in the conclusion s_3, but it does appear in each of the premises. This third term is called the middle term.

Each term of a syllogism may be assigned to a particular set. In this article the set C_3 will correspond to the major term, C_1 will correspond to the minor term, and C_2 will correspond to the middle term.

The seventh example of a categorical syllogism considered in section 7 is the following:

s_1: All engineers are pragmatic.
s_2: Some engineers are wealthy.
$\therefore\ s_3$: Some wealthy persons are pragmatic.

The symbol $\therefore$ preceding s_3, means "therefore".

According to the above explanations, the sets corresponding to the different terms of that syllogism are the following:

C_1: wealthy persons C_2: engineers C_3: pragmatic persons

Given these conventions, the syllogism under consideration can be presented as follows:

s_1: All C_2 are C_3.
s_2: Some C_2 are C_1.
$\therefore$ s_3: Some C_1 are C_3.

6 Specification of the Membership Table Method (MTM)

With the terminology of propositional calculus, the most important characteristic of each categorical syllogism – that s_3 may be inferred, or deduced, from s_1 and s_2 – can be expressed as follows: $(s_1 \wedge s_2) \rightarrow s_3$.

The internal structure of the propositions s_1, s_2 and s_3 is not made evident by their names. It is thus not feasible to use a resource directly from propositional calculus, such as a truth table to prove in the case of the syllogism considered that it is valid that $(s_1 \wedge s_2) \rightarrow s_3$ is a tautology, or law, of that calculus. However, the proposition $(s_1 \wedge s_2) \rightarrow s_3$ does make it possible to show one of the most relevant characteristics of valid categorical syllogisms: If $(s_1 \wedge s_2)$ – that is, the conjunction of s_1 and s_2 which is the antecedent of that proposition – is true, then s_3 (the consequent of that proposition) is true.

According to section 2, a certain set corresponds to each proposition. The sets corresponding respectively to s_1, s_2 and s_3 will be denominated S_1, S_2 and S_3. Each of those sets does reveal the relation between the terms of the corresponding proposition.

In addition, according to section 2, the set $(S_1 \cap S_2) \twoheadrightarrow S_3$ corresponds to the proposition $s_1 \wedge s_2) \rightarrow s_3$. That set will be denominated S: $S = (S_1 \cap S_2) \twoheadrightarrow S_3$.

According to the MTM, the corresponding set S should be obtained for each categorical syllogism. If S is equal to the universal set $\mathbb{U}$ – that is, if $S = \mathbb{U}$ – then that categorical syllogism is valid. If $S \neq \mathbb{U}$, then that syllogism is not valid. The different types of invalid categorical syllogisms will not be considered in this article. The MTM will be used only to determine whether the categorical syllogism considered is valid or not valid.

Recall that in the column for S – the set corresponding to the syllogism considered – in the membership table of that set, the following should be examined:

a) If each element of that column is equal to one (1) then $S = \mathbb{U}$, and the syllogism considered is, therefore, valid;
b) If at least one element of that column is equal to zero (0), then $S \neq \mathbb{U}$, and the syllogism considered is, therefore, not valid.

7 Determining the Validity, or the Non-Validity, of Diverse Categorical Syllogisms Using the MTM: Examples

The first six examples of the categorical syllogisms considered in this section were taken from [17].

For each example considered, the following aspects are specified: 1) the categorical syllogism expressed in natural language; 2a) that syllogism expressed in terms of the sets C_1, C_2 and C_3, and 2b) the corresponding sets S_1, S_2 and S_3, respectively; 3) the set S corresponding to the syllogism; and 4) the membership table which makes it possible to determine whether $S = \mathbb{U}$, in which case the syllogism is valid, or whether $S \neq \mathbb{U}$, in which case the syllogism is not valid.

Example 1

s_1: All men are mortal.
s_2: All Greeks are men.
$\therefore s_3$: All Greeks are mortal.

C_1: Greeks $\quad$ C_2: men $\quad$ C_3: mortal beings

s_1: All C_2 are C_3. $\quad$ $S_1 = (C_2 \twoheadrightarrow C_3)$
s_2: All C_1 are C_2. $\quad$ $S_2 = (C_1 \twoheadrightarrow C_2)$
$\therefore s_3$: All C_1 are C_3. $\quad$ $S_3 = (C_1 \twoheadrightarrow C_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_2 \twoheadrightarrow C_3) \cap (C_1 \twoheadrightarrow C_2)) \twoheadrightarrow (C_1 \twoheadrightarrow C_3)$$

C_1	C_2	C_3	$S_1 =$ $C_2 \twoheadrightarrow C_3$	$S_2 =$ $(C_1 \twoheadrightarrow C_2)$	$S_3 =$ $(C_1 \twoheadrightarrow C_3)$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	0	1	0	1
1	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

Figure 9: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 2

s_1: No men are perfect.
s_2: All Greeks are men.
$\therefore\, s_3$: No Greeks are perfect.

C_1: Greeks $\qquad$ C_2: men $\qquad$ C_3: perfect beings

s_1: No C_2 are C_3.	$S_1 = (C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C}_3)$
s_2: All C_1 are C_2.	$S_2 = (C_1 \mathrel{-\!\!+\!\!\rightarrow} C_2)$
$\therefore\, s_3$: No C_1 are C_3.	$S_3 = (C_1 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \mathrel{-\!\!+\!\!\rightarrow} S_3) = ((C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C}_3) \cap (C_1 \mathrel{-\!\!+\!\!\rightarrow} C_2)) \mathrel{-\!\!+\!\!\rightarrow} (C_1 \mathrel{-\!\!+\!\!\rightarrow} C_3)$$

C_1	C_2	C_3	$\overset{+}{C}_3$	$S_1 =$ $C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C}_3$	$S_2 =$ $(C_1 \mathrel{-\!\!+\!\!\rightarrow} C_2)$	$S_3 =$ $(C_1 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \mathrel{-\!\!+\!\!\rightarrow} S_3$
0	0	0	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	0	1
1	0	0	1	1	0	1	0	1
1	0	1	0	1	0	0	0	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	0	0	1

Figure 10: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 3

s_1: All philosophers are wise.
s_2: Some Greeks are philosophers.
$\therefore\ s_3$: Some Greeks are wise.

C_1: Greeks $\qquad$ C_2: philosophers $\qquad$ C_3: wise persons

s_1: All C_2 are C_3	$S_1 = (C_2 \twoheadrightarrow C_3)$
s_2: Some C_1 are C_2	$S_2 = (C_1 \cap C_2)$
$\therefore\ s_3$: Some C_1 are C_3	$S_3 = (C_1 \cap C_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_2 \twoheadrightarrow C_3) \cap (C_1 \cap C_2)) \twoheadrightarrow (C_1 \twoheadrightarrow C_3)$$

C_1	C_2	C_3	$S_1 = C_2 \twoheadrightarrow C_3$	$S_2 = (C_1 \cap C_2)$	$S_3 = (C_1 \cap C_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	0	0	0	1
0	0	1	1	0	0	0	1
0	1	0	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	1	0	0	0	1
1	0	1	1	0	1	0	1
1	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

Figure 11: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 4

s_1: No philosophers are wicked.
s_2: Some Greeks are philosophers.
$\therefore s_3$: Some Greeks are not wicked.

C_1: Greeks $\qquad$ C_2: philosophers $\qquad$ C_3: wicked persons

s_1: No C_2 are C_3. $\qquad$ $S_1 = (C_2 \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_3)$
s_2: Some C_1 are C_2. $\qquad$ $S_2 = (C_1 \cap C_2)$
$\therefore s_3$: Some C_1 are not C_3. $\qquad$ $S_3 = (C_1 \cap \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \mathrel{+\!\!\!\!\rightarrow} S_3) = ((C_2 \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_3) \cap (C_1 \cap C_2)) \mathrel{+\!\!\!\!\rightarrow} (C_1 \cap \overset{+}{C}_3)$$

C_1	C_2	C_3	$\overset{+}{C}_3$	$S_1 = C_2 \mathrel{+\!\!\!\!\rightarrow} \overset{+}{C}_3$	$S_2 = (C_1 \cap C_2)$	$S_3 = (C_1 \cap \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \mathrel{+\!\!\!\!\rightarrow} S_3$
0	0	0	1	1	0	0	0	1
0	0	1	0	1	0	0	0	1
0	1	0	1	1	0	0	0	1
0	1	1	0	0	0	0	0	1
1	0	0	1	1	0	1	0	1
1	0	1	0	1	0	0	0	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	0	0	1

Figure 12: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 5

s_1: All Greeks are men.
s_2: Some mortals are not men.
$\therefore s_3$: Some mortals are not Greeks.

C_1: mortal beings $\quad$ C_2: men $\quad$ C_3: Greeks

s_1: All C_3 are C_2. $\quad$ $S_1 = (C_3 \mathrel{+\!\!\!\!\!\longrightarrow} C_2)$

s_2: Some C_1 are not C_2. $\quad$ $S_2 = (C_1 \cap \overset{+}{C}_2)$

$\therefore s_3$: Some C_1 are not C_3. $\quad$ $S_3 = (C_1 \cap \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \mathrel{+\!\!\!\!\!\longrightarrow} S_3) = ((C_3 \mathrel{+\!\!\!\!\!\longrightarrow} C_2) \cap (C_1 \cap \overset{+}{C}_2)) \mathrel{+\!\!\!\!\!\longrightarrow} (C_1 \cap \overset{+}{C}_3)$$

C_1	C_2	C_3	$\overset{+}{C}_2$	$\overset{+}{C}_3$	$S_1 = C_3 \mathrel{+\!\!\!\!\!\longrightarrow} C_2)$	$S_2 = (C_1 \cap \overset{+}{C}_2)$	$S_3 = (C_1 \cap \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \mathrel{+\!\!\!\!\!\longrightarrow} S_3$
0	0	0	1	1	1	0	0	0	1
0	0	1	1	0	0	0	0	0	1
0	1	0	0	1	1	0	0	0	1
0	1	1	0	0	1	0	0	0	1
1	0	0	1	1	1	1	1	1	1
1	0	1	1	0	0	1	0	0	1
1	1	0	0	1	1	0	1	0	1
1	1	1	0	0	1	0	0	0	1

Figure 13: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 6

s_1: Some men are not Greeks.
s_2: All men are mortal.
$\therefore s_3$: Some mortals are not Greeks.

C_1: mortal beings $\quad$ C_2: men $\quad$ C_3: Greeks

s_1: Some C_2 are not C_3. $\quad S_1 = (C_2 \cap \overset{+}{C}_3)$
s_2: All C_2 are C_1. $\quad S_2 = (C_2 \twoheadrightarrow C_1)$
$\therefore s_3$: Some C_1 are not C_3. $\quad S_3 = (C_1 \cap \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_2 \cap \overset{+}{C}_3) \cap (C_2 \twoheadrightarrow C_1)) \twoheadrightarrow (C_1 \cap \overset{+}{C}_3)$$

C_1	C_2	C_3	$\overset{+}{C}_3$	$S_1 = C_2 \cap \overset{+}{C}_3$	$S_2 = (C_2 \twoheadrightarrow C_1)$	$S_3 = (C_1 \cap \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	0	1	0	0	1
0	0	1	0	0	1	0	0	1
0	1	0	1	1	0	0	0	1
0	1	1	0	0	0	0	0	1
1	0	0	1	0	1	1	0	1
1	0	1	0	0	1	0	0	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	0	0	1

Figure 14: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 7

s_1: All engineers are pragmatic.
s_2: Some engineers are wealthy.
$\therefore$ s_3: Some wealthy persons are pragmatic.

C_1: wealthy persons $\quad$ C_2: engineers $\quad$ C_3: pragmatic persons

$$\begin{array}{rll} s_1\text{: All } C_2 \text{ are } C_3. & & S_1 = (C_2 \mathrel{-\!\!\!+\!\!\!\!\rightarrow} C_3) \\ s_2\text{: Some } C_2 \text{ are } C_1. & & S_2 = (C_2 \cap C_1) \\ \therefore s_3\text{: Some } C_1 \text{ are } C_3. & & S_3 = (C_1 \cap C_3) \end{array}$$

$$S = ((S_1 \cap S_2) \mathrel{-\!\!\!+\!\!\!\!\rightarrow} S_3) = ((C_2 \mathrel{-\!\!\!+\!\!\!\!\rightarrow} C_3) \cap (C_2 \cap C_1)) \mathrel{-\!\!\!+\!\!\!\!\rightarrow} (C_1 \cap C_3)$$

If it is taken into account that $(C_1 \cap C_2) = (C_2 \cap C_1)$, it is noted that the set corresponding to the categorical syllogism in example 7 has the same form as the set corresponding to the categorical syllogism in example 3. The latter categorical syllogism is valid. Therefore, the categorical syllogism in example 7 is valid.

Example 8

s_1: No intellectuals are superstitious.
s_2: Some French persons are intellectuals.
$\therefore$ s_3: Some French persons are not superstitious.

C_1: French persons $\quad$ C_2: intellectuals $\quad$ C_3: superstitious persons

$$\begin{array}{rll} s_1\text{: No } C_2 \text{ are } C_3. & & S_1 = (C_2 \mathrel{-\!\!\!+\!\!\!\!\rightarrow} \overset{+}{C}_3) \\ s_2\text{: Some } C_1 \text{ are } C_2. & & S_2 = (C_1 \cap C_2) \\ \therefore s_3\text{: Some } C_1 \text{ are not } C_3. & & S_3 = (C_1 \cap \overset{+}{C}_3) \end{array}$$

$$S = ((S_1 \cap S_2) \mathrel{-\!\!\!+\!\!\!\!\rightarrow} S_3) = ((C_2 \mathrel{-\!\!\!+\!\!\!\!\rightarrow} \overset{+}{C}_3) \cap (C_1 \cap C_2)) \mathrel{-\!\!\!+\!\!\!\!\rightarrow} (C_1 \cap \overset{+}{C}_3)$$

It is noted that the set corresponding to the categorical syllogism in example 8 has the same form as the set corresponding to the categorical syllogism in example 4. The latter categorical syllogism is valid. Therefore, the categorical syllogism in example 8 is valid.

Example 9

s_1: All men are rational.
s_2: All Spaniards are men.
$\therefore s_3$: All Spaniards are rational.

C_1: Spaniards $\quad$ C_2: men $\quad$ C_3: rational men

$$\begin{array}{rlrl} s_1: & \text{All } C_2 \text{ are } C_3. & \qquad S_1 = & (C_2 \twoheadrightarrow C_3) \\ s_2: & \text{All } C_1 \text{ are } C_2. & \qquad S_2 = & (C_1 \twoheadrightarrow C_2) \\ \therefore s_3: & \text{All } C_1 \text{ are } C_3. & \qquad S_3 = & (C_1 \twoheadrightarrow C_3) \end{array}$$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_2 \twoheadrightarrow C_3) \cap (C_1 \twoheadrightarrow C_2)) \twoheadrightarrow (C_1 \twoheadrightarrow C_3)$$

It is noted that the set corresponding to the categorical syllogism in example 9 has the same form as the set corresponding to the categorical syllogism in example 1. The latter categorical syllogism is valid. Therefore, the categorical syllogism in example 9 is valid.

Example 10

s_1: All sculptors are artists.
s_2: No artists are fossils.
$\therefore s_3$: No fossils are sculptors.

C_1: fossils C_2: artists C_3: sculptors

s_1: All C_3 are C_2.	$S_1 = (C_3 \mathrel{-\!\!+\!\!\rightarrow} C_2)$
s_2: No C_2 are C_1.	$S_2 = (C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_1})$
$\therefore s_3$: No C_1 are C_2.	$S_3 = (C_1 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_2})$

$$S = ((S_1 \cap S_2) \mathrel{-\!\!+\!\!\rightarrow} S_3) = ((C_3 \mathrel{-\!\!+\!\!\rightarrow} C_2) \cap (C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_1})) \mathrel{-\!\!+\!\!\rightarrow} (C_1 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_2})$$

C_1	C_2	C_3	$\overset{+}{C_1}$	$\overset{+}{C_2}$	$S_1 =$ $C_3 \mathrel{-\!\!+\!\!\rightarrow} C_2$	$S_2 =$ $(C_2 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_1})$	$S_3 =$ $(C_1 \mathrel{-\!\!+\!\!\rightarrow} \overset{+}{C_2})$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \mathrel{-\!\!+\!\!\rightarrow} S_3$
0	0	0	1	1	1	1	1	1	1
0	0	1	1	1	0	1	1	0	1
0	1	0	1	0	1	1	1	1	1
0	1	1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	1	0	1	0	1	1	0	1
1	1	0	0	0	1	0	0	0	1
1	1	1	0	0	1	0	0	0	1

Figure 15: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 11

s_1: No humanists are corrupt.
s_2: All despots are corrupt.
$\therefore s_3$: No despots are humanists.

C_1: despots $\qquad$ C_2: corrupt persons $\qquad$ C_3: humanists

s_1: No C_3 are C_2.	$S_1 = (C_3 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_2)$
s_2: All C_1 are C_2.	$S_2 = (C_1 \mathrel{\rlap{\;\;+}\longrightarrow} C_2)$
$\therefore s_3$: No C_1 are C_3.	$S_3 = (C_1 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \mathrel{\rlap{\;\;+}\longrightarrow} S_3) = ((C_3 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_2) \cap (C_1 \mathrel{\rlap{\;\;+}\longrightarrow} C_2)) \mathrel{\rlap{\;\;+}\longrightarrow} (C_1 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_3)$$

C_1	C_2	C_3	$\overset{+}{C}_2$	$\overset{+}{C}_3$	$S_1 =$ $C_3 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_2$	$S_2 =$ $(C_1 \mathrel{\rlap{\;\;+}\longrightarrow} C_2)$	$S_3 =$ $(C_1 \mathrel{\rlap{\;\;+}\longrightarrow} \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \mathrel{\rlap{\;\;+}\longrightarrow} S_3$
0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1
0	1	0	0	1	1	1	1	1	1
0	1	1	0	0	0	1	1	0	1
1	0	0	1	1	1	0	1	0	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	1	1	1	1	1	1
1	1	1	0	0	0	1	0	0	1

Figure 16: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 12

s_1: Some mammals are dogs.
s_2: All mammals are vertebrates.
$\therefore$ s_3: Some vertebrates are dogs.

C_1: vertebrates $\qquad$ C_2: mammals $\qquad$ C_3: dogs

s_1: Some C_2 are C_3. $\qquad$ $S_1 = (C_2 \cap C_3)$
s_2: All C_2 are C_1. $\qquad$ $S_2 = (C_2 \twoheadrightarrow C_1)$
$\therefore$ s_3: Some C_1 are C_3. $\qquad$ $S_3 = (C_1 \cap C_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_2 \cap C_3) \cap (C_2 \twoheadrightarrow C_1)) \twoheadrightarrow (C_1 \cap C_3)$$

C_1	C_2	C_3	$S_1 = C_2 \cap C_3$	$S_2 = (C_2 \twoheadrightarrow C_1)$	$S_3 = (C_1 \cap C_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1
0	1	0	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

Figure 17: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 13

s_1: No artists are Neo-Kantians.
s_2: Some Germans are Neo-Kantians.
$\therefore\ s_3$: Some Germans are not artists.

C_1: Germans $\quad$ C_2: Neo-Kantians $\quad$ C_3: artists

s_1: No C_3 are C_2. $\quad$ $S_1 = (C_3 \twoheadrightarrow \overset{+}{C}_2)$
s_2: Some C_1 are C_2. $\quad$ $S_2 = (C_1 \cap C_2)$
$\therefore\ s_3$: Some C_1 are not C_3. $\quad$ $S_3 = (C_1 \cap \overset{+}{C}_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_3 \twoheadrightarrow \overset{+}{C}_2) \cap (C_1 \cap C_2)) \twoheadrightarrow (C_1 \twoheadrightarrow \overset{+}{C}_3)$$

C_1	C_2	C_3	$\overset{+}{C}_2$	$\overset{+}{C}_3$	$S_1 = C_3 \twoheadrightarrow \overset{+}{C}_2$	$S_2 = (C_1 \cap C_2)$	$S_3 = (C_1 \cap \overset{+}{C}_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	1	1	0	0	0	1
0	0	1	1	0	1	0	0	0	1
0	1	0	0	1	1	0	0	0	1
0	1	1	0	0	0	0	0	0	1
1	0	0	1	1	1	0	1	0	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	1	1	1	1	1	1
1	1	1	0	0	0	1	0	0	1

Figure 18: Membership table of the set S corresponding to the syllogism considered

The categorical syllogism considered is valid.

Example 14

s_1: All poets are visionaries.
s_2: All prophets are visionaries.
$\therefore s_3$: Some prophets are poets.

C_1: prophets $\quad$ C_2: visionaries $\quad$ C_3: poets

s_1: All C_3 are C_2.	$S_1 = (C_3 \twoheadrightarrow C_2)$
s_2: All C_1 are C_2.	$S_2 = (C_1 \twoheadrightarrow C_2)$
$\therefore s_3$: Some C_1 are C_3.	$S_3 = (C_1 \cap C_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_3 \twoheadrightarrow C_2) \cap (C_1 \twoheadrightarrow C_2)) \twoheadrightarrow (C_1 \cap C_3)$$

C_1	C_2	C_3	$S_1 =$ $C_3 \twoheadrightarrow C_2$	$S_2 =$ $(C_1 \twoheadrightarrow C_2)$	$S_3 =$ $(C_1 \cap C_3)$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	1	0	1	0
0	0	1	0	1	0	0	1
0	1	0	1	1	0	1	0
0	1	1	1	1	0	1	0
1	0	0	1	0	0	0	1
1	0	1	0	0	1	0	1
1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1

Figure 19: Membership table of the set S corresponding to the syllogism considered

Given that in the column of S there are membership values equal to zero (0), $S \neq \mathbb{U}$. Therefore, the categorical syllogism considered is not valid.

Example 15

s_1: Some landowners are not egotists.
s_2: No philanthropists are egotists.
$\therefore\ s_3$: Some philanthropists are landowners.

C_1: philanthropists C_2: egotists C_3: landowners

s_1: Some C_3 are not C_2. $S_1 = (C_3 \cap \overset{+}{C}_2)$
s_2: No C_1 are C_2. $S_2 = (C_1 \mathrel{-\!\!\!+\!\!\!\to} \overset{+}{C}_2)$
$\therefore s_3$: Some C_1 are C_3. $S_3 = (C_1 \cap C_3)$

$$S = ((S_1 \cap S_2) \mathrel{-\!\!\!+\!\!\!\to} S_3) = ((C_3 \cap \overset{+}{C}_2) \cap (C_1 \mathrel{-\!\!\!+\!\!\!\to} \overset{+}{C}_2)) \mathrel{-\!\!\!+\!\!\!\to} (C_1 \cap C_3)$$

C_1	C_2	C_3	$\overset{+}{C}_2$	$S_1 =$ $C_3 \cap \overset{+}{C}_2$	$S_2 =$ $(C_1 \mathrel{-\!\!\!+\!\!\!\to} \overset{+}{C}_2)$	$S_3 =$ $(C_1 \cap C_3)$	$(S_1 \cap S_2)$	$S =$ $(S_1 \cap S_2) \mathrel{-\!\!\!+\!\!\!\to} S_3$
0	0	0	1	0	1	0	0	1
0	0	1	1	1	1	0	1	0
0	1	0	0	0	1	0	0	1
0	1	1	0	0	1	0	0	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	1
1	1	1	0	0	0	1	0	1

Figure 20: Membership table of the set S corresponding to the syllogism considered

Given that in the column of S there is a membership value equal to zero (0), $S \neq \mathbb{U}$. Therefore, the categorical syllogism considered is not valid.

Example 16

s_1: All cosmologists are scientists.
s_2: Some scientists are polyglots.
$\therefore$ s_3: Some polyglots are cosmologists.

C_1: polyglots $\quad$ C_2: scientists $\quad$ C_3: cosmologists

s_1: All C_3 are C_2. $\quad$ $S_1 = (C_3 \twoheadrightarrow C_2)$
s_2: Some C_2 are C_1. $\quad$ $S_2 = (C_2 \cap C_1)$
$\therefore$ s_3: Some C_1 are C_3. $\quad$ $S_3 = (C_1 \cap C_3)$

$$S = ((S_1 \cap S_2) \twoheadrightarrow S_3) = ((C_3 \twoheadrightarrow C_2) \cap (C_2 \cap C_1)) \twoheadrightarrow (C_1 \cap C_3)$$

C_1	C_2	C_3	$S_1 = C_3 \twoheadrightarrow C_2$	$S_2 = (C_2 \cap C_1)$	$S_3 = (C_1 \cap C_3)$	$(S_1 \cap S_2)$	$S = (S_1 \cap S_2) \twoheadrightarrow S_3$
0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	1	0	0	0	1
1	0	1	0	0	1	0	1
1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1

Figure 21: Membership table of the set S corresponding to the syllogism considered

Given that in the column of S there is a membership value equal to zero (0), $S \neq \mathbb{U}$. Therefore, the categorical syllogism considered is not valid.

8 Discussion and Perspectives

Reference was made to the relation existing between the truth tables of propositional calculus and the membership tables of set theory, used in the MTM, for example. Within the framework of canonical fuzzy logic (CFL) [18], the latter tables can be considered truth tables, in the strict sense. One of the main objectives of this series of articles on logic presented by the authors is to shed light on the relations existing – both in classical logic and in diverse variants of some non-classical logics – between different calculi of these logics, such as propositional calculus and predicate calculus, the last of which is expressed in the terminology of set theory. (Another of the objectives of this research program is to show how classical logic can be considered a "limit case" of those variants of non-classical logics.)

Categorical syllogisms, simple types of reasoning which have been studied a great deal, are important from historical and educational perspectives. In this research program they are a "testbed" for different methods which will be evaluated according to their possibilities of 1) being applied to determine the validity of more complex types of reasoning, and 2) being automatized for their use in disciplines such as control engineering and artificial intelligence. A method different from the MTM to determine the validity of categorical syllogisms will be presented in another article.

Note: A computer program based on the Membership Table Method (MTM) for the determination of the validity of categorical sylllogisms has been published on the followling sites:

1. https://github.com/AppliedMathGroup/Logic
2. https://www.appliedmathgroup.org/en/mtm.htm

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